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20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

A method is derived to compute the force signature of a body impacting on a linear elastic structure. The time history of the impacting force is reconstructed from the response to impulse and response to the force at various points on the structure. The method is quite general and is applicable to any linear structure. The method is tested on a two degree of freedom system with various applied forces and reconstructs the forces very accurately.

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CONTENTS

A METHOD TO COMPUTE THE FORCE SIGNATURE OF A BODY IMPACTING ON A LINEAR ELASTIC STRUCTURE USING FOURIER ANALYSIS

INTRODUCTION

NRL has historically been involved in defining shock design inputs for equipment aboard submarines. A body, such as a torpedo, impacting on a structure, such as a submarine hull, is one subset of this general class. This report presents a general method of determining the force signature of a body impacting on any linear elastic structure.

Consider equipment which has been attached to a structure, for example a submarine hull, that is struck by a body such as an inert torpedo at one of the frames. The goal is to determine the force exerted by the body impacting on the structure by using the measured response at various gages on the equipment. In addition the impulse response at various gage locations is needed. The response to impulse may be obtained by using a standard computer structural analysis code such as NASTRAN.

ANALYSIS

Consider responses recorded by gages at several points on the equipment. For a linear elastic structure the response at point P due to a force at point K is the convolution of the force at point K and the response at point P due to a unit impulse at point K. This is expressed as:

$$R_P(t) = \int_0^t F_K(T)\, I_{PK}(t - T)\, dT \tag{1}$$

or:

$$R_P = F_K * I_{PK}. \tag{2}$$

The convolution of two transforms in time domain is the inverse transform of the product in frequency domain. Thus

$$R_P(\omega) = F_K(\omega)\, I_{PK}(\omega) \tag{3}$$

where $R_P(\omega)$, $F_K(\omega)$, $I_{PK}(\omega)$ are the Fourier transforms of $R_P(t)$, $F_K(t)$, and $I_{PK}(t)$. $R_P(\omega)$ for example is defined by:

$$R_P(\omega) = \int_{-\infty}^{+\infty} R_P(t) e^{-i\omega t}\, dt \tag{4}$$

and its inverse transform by:

$$R_P(t) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} R_P(\omega) e^{i\omega t}\, d\omega. \tag{5}$$

In order to make use of a very accurate numerical method to compute Fourier sine and cosine transforms derived in a previous report [1], it is necessary to express the transforms in Eq. (3) in terms of sine and cosine transforms and find the associated inverse transforms. Consider a general function $f(p)$ which is real and satisfies the Dirichlet conditions and the following integral exists

$$\int_{-\infty}^{+\infty} |f(p)|\, dp. \tag{6}$$

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Writing the Fourier transform of $f(p)$ as:

$$F(\omega) = \int_{-\infty}^{+\infty} f(p)e^{-i\omega p}\, dp = \int_{-\infty}^{+\infty} f(p) \cos \omega p\, dp - i\int_{-\infty}^{+\infty} f(p) \sin \omega p\, dp \tag{7}$$

and defining

$$F(\omega) = F_C(\omega) - i\, F_S(\omega). \tag{8}$$

Writing the inverse Fourier transform of $F(\omega)$ as:

$$f(t) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} F(\omega)e^{i\omega t} d\omega = \frac{1}{2\pi} \int_{-\infty}^{+\infty} (F_C - i\, F_S)\, (\cos \omega t + i \sin \omega t)\, d\omega \tag{9}$$

then rewriting

$$\begin{aligned} f(t) &= \frac{1}{2\pi} \int_{-\infty}^{+\infty} F_C \cos \omega t\, d\omega + \frac{1}{2\pi} \int_{-\infty}^{+\infty} F_S \sin \omega t\, d\omega \\ &- \frac{i}{2\pi} \int_{-\infty}^{+\infty} F_S \cos \omega t\, d\omega + \frac{i}{2\pi} \int_{-\infty}^{+\infty} F_C \sin \omega t\, d\omega. \end{aligned} \tag{10}$$

Since $F_S(\omega) = -F_S(-\omega)$, F_S is an odd function, and since $\cos \omega t$ is an even function the first integrand on the second line of Eq. (10) is odd; because the integral is over the symmetric limits $-\infty$ to $+\infty$ it vanishes. The second integrand on the second line is also odd and therefore both imaginary integrals vanish. The integrands on the top line of Eq. (10) are even and consequently:

$$f(t) = \frac{1}{\pi} \int_0^{\infty} F_C \cos \omega t\, d\omega + \frac{1}{\pi} \int_0^{\infty} F_S \sin \omega t\, d\omega. \tag{11}$$

Since for a physical system $f(p) = 0$ for $p < 0$ $F_C(\omega)$ and $F_S(\omega)$ become

$$F_C(\omega) = \int_0^{\infty} f(p) \cos \omega p\, dp \tag{12}$$

$$F_S(\omega) = \int_0^{\infty} f(p) \sin \omega p\, dp \tag{13}$$

where F_C and F_S are now the familiar Fourier cosine and sine transforms. If $f(p)$ is an even function, from Eqs. (7) and (9):

$$f(t) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} f(p) \cos \omega p \cos \omega t\, dp\, d\omega \tag{14}$$

which may be written as:

$$f_C(t) = \frac{2}{\pi} \int_0^{\infty} F_C(\omega) \cos \omega t\, d\omega \tag{15}$$

and is the inverse Fourier cosine transform. This transform and the Fourier cosine transform:

$$F_C(\omega) = \int_0^{\infty} f_C(p) \cos \omega p\, dp \tag{16}$$

make up a convenient transform pair. If $f(p)$ is an odd function from Eqs. (7) and (9)

$$f(t) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} f(p) \sin \omega p \sin \omega t\, dp\, d\omega \tag{17}$$

which may be written as:

$$f_S(t) = \frac{2}{\pi} \int_0^{\infty} F_S(\omega) \sin \omega t\, d\omega \tag{18}$$

and is the inverse Fourier sine transform. This transform and the Fourier sine transform:

$$F_S(\omega) = \int_0^{\infty} f_S(p) \sin \omega p\, dp \tag{19}$$

make up a transform pair. Returning to Eq. (3) and writing in terms of sine and cosine transforms

$$(R_C - i\, R_S)_P = (F_C - i\, F_S)_K\, [I_C - iI_S]_{PK}. \tag{20}$$

Dropping the subscripts for convenience when dealing with only one response point in this derivation, yields, when setting real and imaginary parts equal:

$$R_C = F_C I_C - F_S I_S \tag{21}$$

and

$$R_S = F_C I_S + F_S I_C; \tag{22}$$

where R_C is the Fourier cosine transform of the response,
R_S is the Fourier sine transform of the response,
I_C is the Fourier cosine transform of the computer developed response to impulse,
I_S is the Fourier sine transform of the computer developed resonse to impulse.

Note this is impulsive response at P due to unit impulse at K when R is response at P and the impact force is at K.

Solving for the Fourier transforms F_C and F_S yields

$$F_C = \frac{R_C I_C + R_S I_S}{I_C^2 + I_S^2} = F_C(\omega), \text{ Fourier cosine transform} \tag{23}$$

and,

$$F_S = \frac{R_S I_C - R_C I_S}{I_C^2 + I_S^2} = F_S(\omega); \text{ Fourier sine transform,} \tag{24}$$

functions of ω.

For a forcing function $f(t)$ undefined for $t < 0$ one need not consider $f(t)$ before $t = 0$ and may choose $f(t)$ to be even, odd, or neither. The impact force may then be obtained from Eqs. (11), (15) or (18). Typically one would compute both $F_S(\omega)$ and $F_C(\omega)$ and depending on which transform converged toward zero faster choose either the inverse cosine transform (Eq. (15)) or inverse sine transform (Eq. (18)) to get the time history of the impact force.

The process is then repeated for the zth point, etc., and a set of estimates can be built up. The spread present in them is then a measure of how well the computer model of the structure-equipment fits the real world.

COMPUTATION OF FOURIER SINE AND COSINE TRANSFORMS AND INVERSE TRANSFORMS

In Ref. [1] it is shown that the Fourier cosine and sine transforms computed over a finite time T_0 of a function $f(t)$ expressed as:

$$F_C(\omega) = \int_0^{T_0} f(t) \cos \omega t \, dt \tag{25}$$

$$F_S(\omega) = \int_0^{T_0} f(t) \sin \omega t \, dt \tag{26}$$

may be computed from:

$$F_C(\omega) = -X(T_0) \cos \omega T_0 + \frac{\dot{X}(T_0) + f(T_0)}{\omega} \sin \omega T_0 \tag{27}$$

$$F_S(\omega) = -\frac{\dot{X}(T_0) + f(T_0)}{\omega} \cos \omega T_0 - X(T_0) \sin \omega T_0 \tag{28}$$

where X is the displacement response of a linear oscillator

$\dot{X}$ is the velocity response of a linear oscillator

ω is the circular frequency (rads/s)

T_0 is the end of the time interval

The problem of finding the transforms resolves itself into finding $X(T_0)$ and $\dot{X}(T_0)$ for a given value of ω. T_0 and $f(T_0)$ are known. The process is repeated for a number of evenly spaced values of ω over the desired range. The method to find $X(T_0)$ and $\dot{X}(T_0)$ is the numerical integration scheme presented in [1] but with some trigonometric substitutions to prevent computer precision problems. The integration scheme is as follows:

$$X_{n+1} = X_n \cos \omega h + \frac{\dot{X}_n \sin \omega h}{\omega} - \frac{2S_n \sin^2 (\omega h/2)}{\omega^2 h} - SS_{n-1} \left[\frac{\cos^2(\omega h/2)}{\omega^2 h} - \frac{\sin \omega h}{\omega^3 h^2} \right] \tag{29}$$

$$\dot{X}_{n+1} = -\omega X_n \sin \omega h + \dot{X}_n \cos \omega h - S_n \frac{\sin \omega h}{\omega h} - SS_{n-1} \left[\frac{2 \sin^2 (\omega h/2)}{\omega^2 h^2} - \frac{\sin \omega h}{2\omega h} \right] \tag{30}$$

where h is the time increment, and $S_n = f_{n+1} - f_n$ and $SS_{n-1} = f_{n+1} - 2f_n + f_{n-1}$.

The inverse Fourier cosine and sine transforms, defined by:

$$f_C(t) = \frac{2}{\pi} \int_0^{W_0} F_C(\omega) \cos \omega t \, d\omega \tag{31}$$

$$f_S(t) = \frac{2}{\pi} \int_0^{W_0} F_S(\omega) \sin \omega t \, d\omega \tag{32}$$

may be computed in an exactly analogous way by letting ω become t, T_0 become W_0, and h becoming the increment in ω in Eqs. (29) and (30) and multiplying the final result by $2/\pi$.

The numerical integration method has two types of error present: inherent and round off. For transforms involving functions which have a finite number of finite discontinuities and can be exactly described by a set of straight lines or parabolic arcs the method has no inherent error. The numerical integration equations for the case above will give exactly the transform value except for round off error at any ω regardless of the increment size $h(\Delta t)$. For other functions the closer the approximating curve lies to the true function the more exact the transform.

An alternative method to compute Fourier transforms is the fast Fourier transform or FFT. Some drawbacks are that with the FFT the function must be long enough to get the desired frequency increment $\Delta\omega$ and the number of frequencies or samples must be a power of 2. With this method one simply chooses a $\Delta\omega$ and the number of frequencies desired; or one may compute the transform at any desired frequency. With the FFT the frequencies are restricted to multiples of $\Delta\omega$ and one cannot obtain transform values at frequencies in between samples except by decreasing the sample spacing or interpolation.

TWO DEGREE OF FREEDOM SYSTEM TO TEST FORCES SIGNATURE RECONSTRUCTION

In order to test the method to reconstruct the force signature, a 2 degree of freedom model as shown in Fig. 1 consisting of 2 coupled mass-spring-damper systems is employed. The second mass is driven by a force of known shape and the response of both masses are calculated to this force. In addition the response of the masses to a unit impulse is also calculated. Knowing the response to a force and response to impulse a force time history may be reconstructed. Estimates for the force time history can be obtained from both masses. Input forces of various shapes such as parabolic, damped sine, and ramp are tested.

The 2-degree of freedom system in Fig. 1 is governed by the two coupled second order differential equations

$$M_2\ddot{y}_2 + C_2\dot{y}_2 + K_2(y_2 - y_1) = F(t) \tag{33}$$

$$M_1\ddot{y}_1 + C_1\dot{y}_1 + K_2(y_1 - y_2) + k_1 y_1 = 0 \tag{34}$$

For the following test cases the following parameters are used:*

$M_2 = 1$ lb-s²/in	$K_2 = 27000$ lbs/in	$C_2 = 100$ lb-s/in
$M_1 = 2$ lb-s²/in	$K_1 = 9000$ lbs/in	$C_1 = 200$ lb-s/in

Case I: Parabolic Force

A parabolic input force of form:

$$F(t) = 16000\,(t - 40\,t^2) \qquad 0 < t \leqslant .025 \text{ s}$$
$$F(t) = 0 \qquad t > .025 \text{ s}$$

as shown in Fig. 2a is applied. The exact analytical solution is obtained for y_1 and y_2 by the method of Laplace transforms. The solution is straight forward but lengthy and will not be shown here.

The solution for y_1 and y_2 where the force is a unit impulse is also obtained by Laplace transforms. Using a computer program, the sine and cosine transforms of the response of y_1 to impulse and response to the parabolic force are computed from Eqs. (27-30) where the end of the time interval T_0 is .3 s, h the time increment is .0025 s and the frequency ω ranges from 0 to 2000 rads/s in increments of 5 rads/s. These transforms are then combined according to Eqs. (23) and (24) to obtain the sine and cosine transforms of the force shown in Figs. 2b and 2c. The inverse transforms are obtained using the same equations as for the forward transforms Eqs. (27-30) where the end of the frequency interval W_0 is 2000 rads/s, $\Delta\omega$ is 5 rad/s and the time ranges from 0 to .1 s in increments of .0025 s. The inverse transforms are combined according to Eq. (11) to obtain the force time history shown in Fig. 2d. By comparing Figs. 2a and 2d, it can be seen that the input force and the reconstructed force agree very well. The same agreement is obtained if the response of y_2 is used to reconstruct the force. It should be noted that in order to reconstruct the force out to time t it is necessary to have the responses out to a latter time $t + T$. For example, in the case above the impulse response and response to force were used out to .3 s to reconstruct the force time history out to .1 s.

Case II: Down Ramp Force

A down ramp input force of form:

$$F(t) = 100. - (100/.025)\,t \qquad 0 < t \leqslant .025 \text{ s}$$
$$F(t) = 0 \qquad t > .025 \text{ s}$$

as shown in Fig. 3a is applied. In order to avoid recalculating long analytical solutions for each input force a fourth order Runge-Kutta numerical method is used to compute the response to the above force; however the analytical solution for the impulse response is used. Transforms of the same length and increment size as in Case I are taken and the same procedure is used to obtain the force transforms shown in Figs. 3b and 3c. The cosine transform as expected converges much faster than the sine transform. Figure 3d shows the result of using both the sine and cosine transform to reconstruct the input force. Figure 3e shows an improved result by using only the cosine transform to reconstruct the input force. This improvement by using only the cosine transform is for the following reason. If the

*Metric conversion factors are .0254 m/in, 4.4482 N/lb, and 175.13 kg/(lb-s²/in) reflection of the down-ramp function around the y axis is considered, the function becomes even and so only the cosine transform is required in the inverse transform. When the inverse transform is taken by using Eq. (15), the cosine transform, because it converges faster than the sine transform reconstructs the function for $t \geqslant 0$ more accurately than both transforms together as in Eq. 11. Since the forcing function doesn't exist before $t = 0$ the inverse transform should not be considered before $t = 0$; in the above case the ramp before $t = 0$. For an unknown force both the sine and cosine transforms may be examined and the one which converges faster may be used for the inverse transform for $t \geqslant 0$.

Case III: Damped Sine Force

A damped sine wave input force of form:

$$F(t) = (1. - t/T) \sin 4\pi t/T \qquad 0 < t \leqslant .025 \text{ s}, \; T = .025 \text{ s}$$
$$F(t) = 0 \qquad t > .025 \text{ s}$$

as shown in Fig. 4a is applied. As in Case II the response to the force is calculated using a Runge-Kutta method. The same procedure as with Case II is used to calculate the sine and cosine transform of the force, shown in Figs. 4b and 4c. The force reconstructed by using both transforms is shown in Fig. 4d.

Case IV: Exponential Decaying Force

A decaying exponential input force of form:

$$F(t) = 100 \, e^{-500t} \qquad 0 < t < \infty$$

as shown in Fig. 5a is applied. As in Case III the response to the force is calculated using a Runge-Kutta method. The same procedure as in Case III is used to calculate the sine and cosine transform of the force shown in Figs. 5b and 5c. The force reconstructed by using both transforms is shown in Fig. 5d. The force reconstructed by the cosine transform above shown in Fig. 5e is closer to the input force because the cosine transform converges faster than the sine transform.

SUMMARY

An accurate method has been presented to compute the force signature of a body impacting on a linear elastic structure. The various input forces applied to a two degree of freedom system were all reconstructed with good accuracy, and assuming that that accurate impulse responses could be obtained the method would work equally as well on a complex multi-degree of freedom structure.

Reference

1. O'Hara, G.J., "A Numerical Procedure for Shock and Fourier Analysis," NRL Report 5772, June 1962.

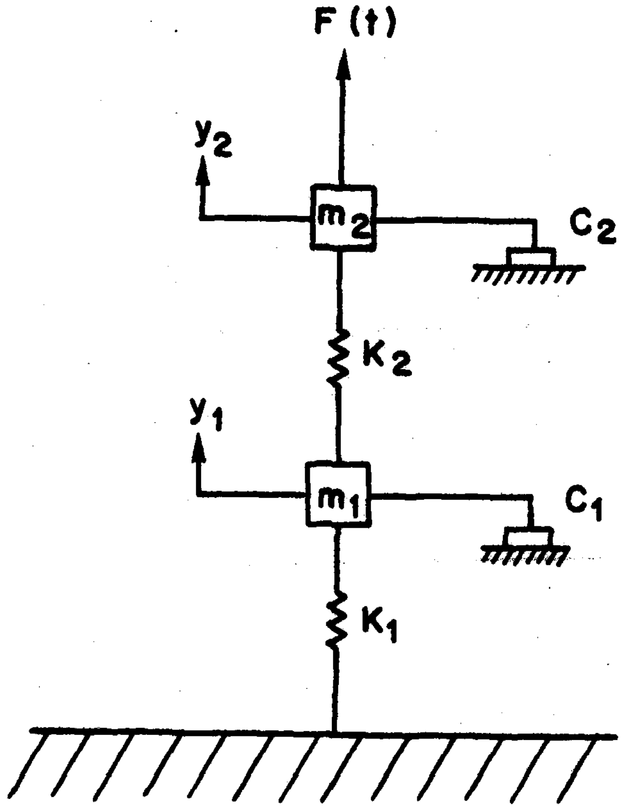

Fig. 1 — Two degree of freedom system to test force signature reconstruction

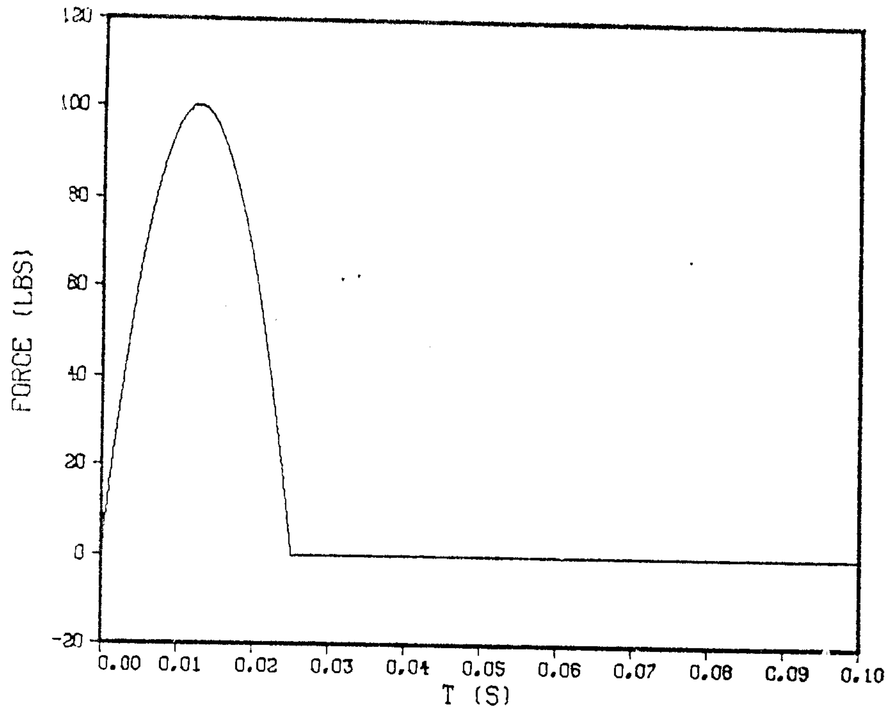

Fig. 2a — Parabolic input force

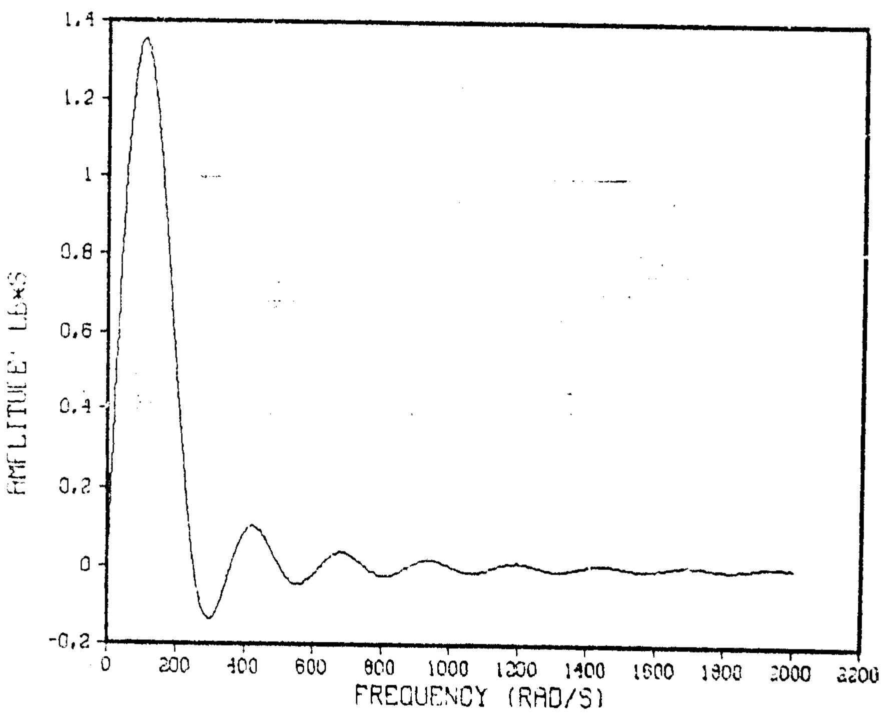

Fig. 2b — Sine transform of reconstructed force

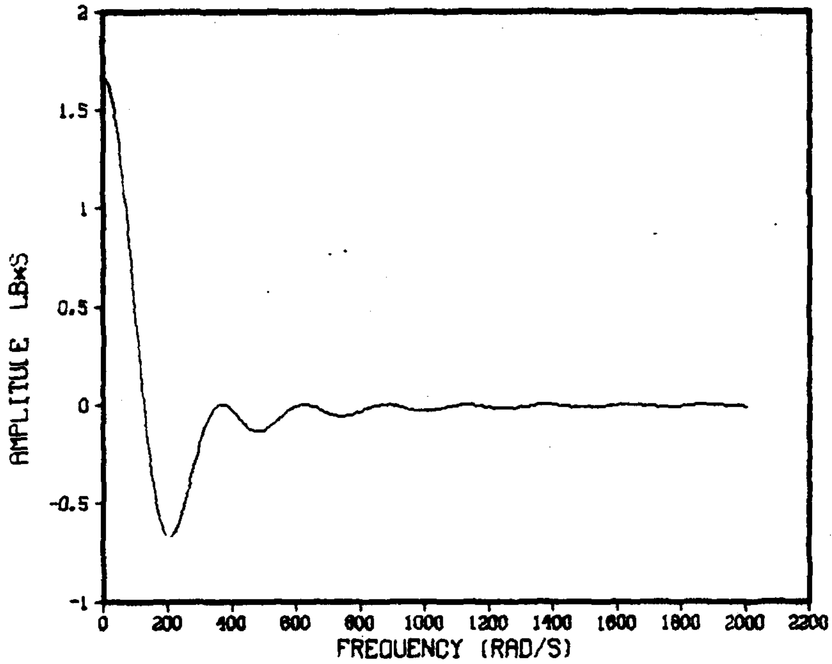

Fig. 2c — Cosine transform of reconstructed force

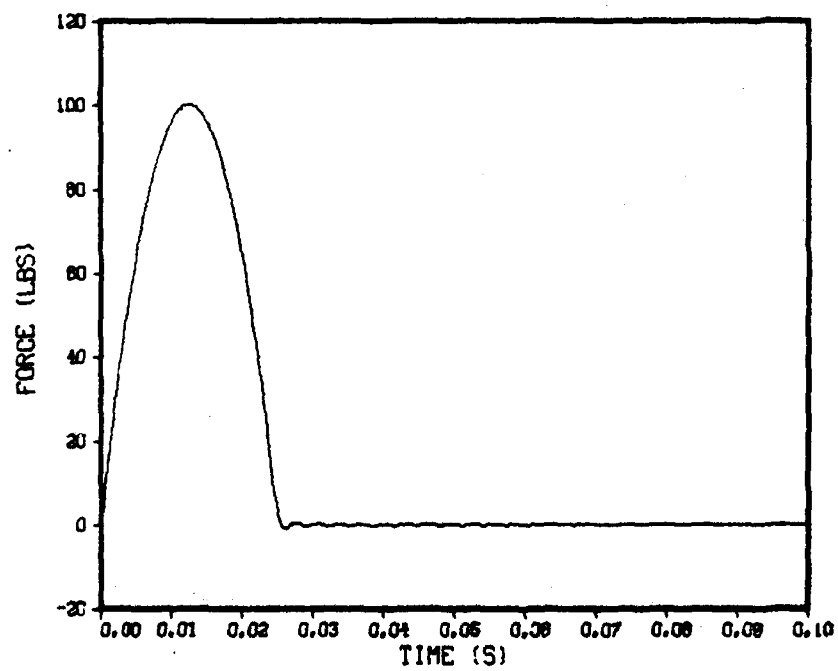

Fig. 2d — Reconstructed input force

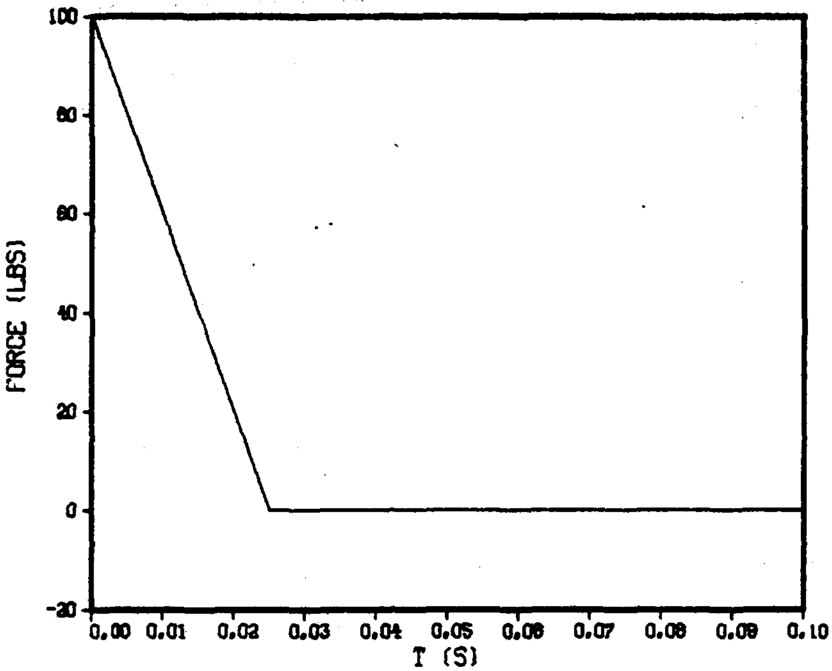

Fig. 3a — Down-ramp input force

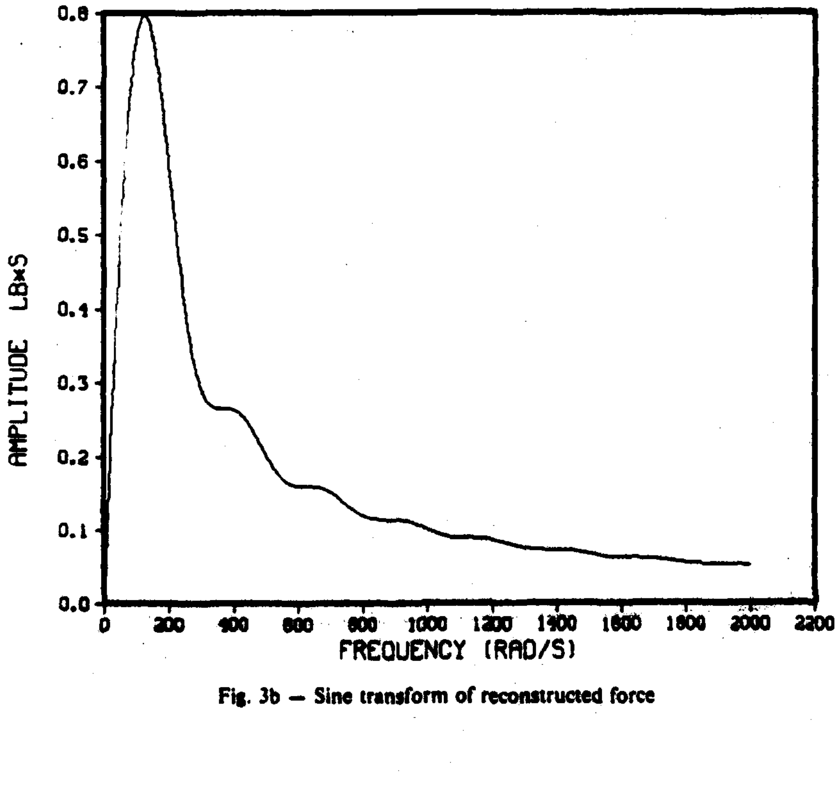

Fig. 3b — Sine transform of reconstructed force

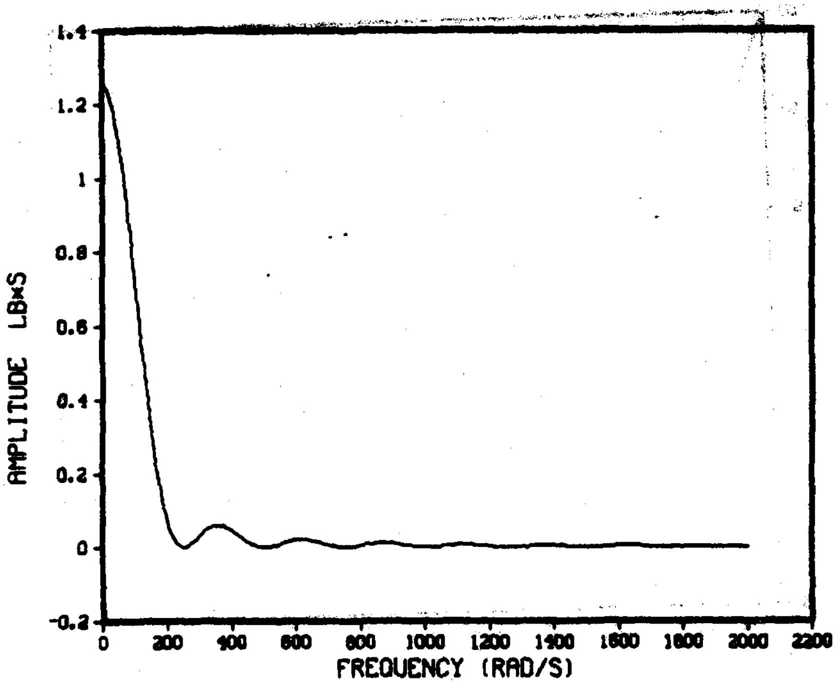

Fig. 3c — Cosine transform of reconstructed force

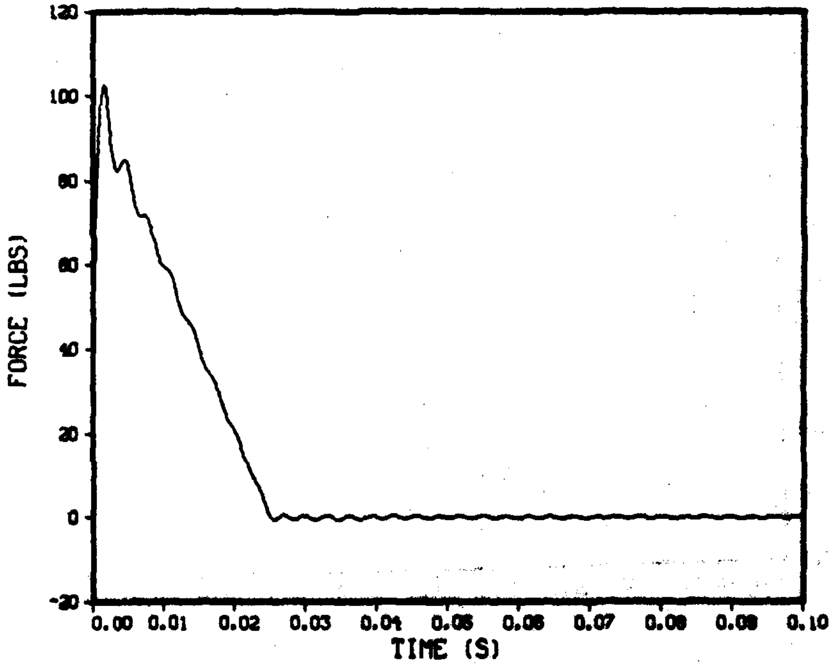

Fig. 3d — Reconstructed input force using both sine and cosine transforms

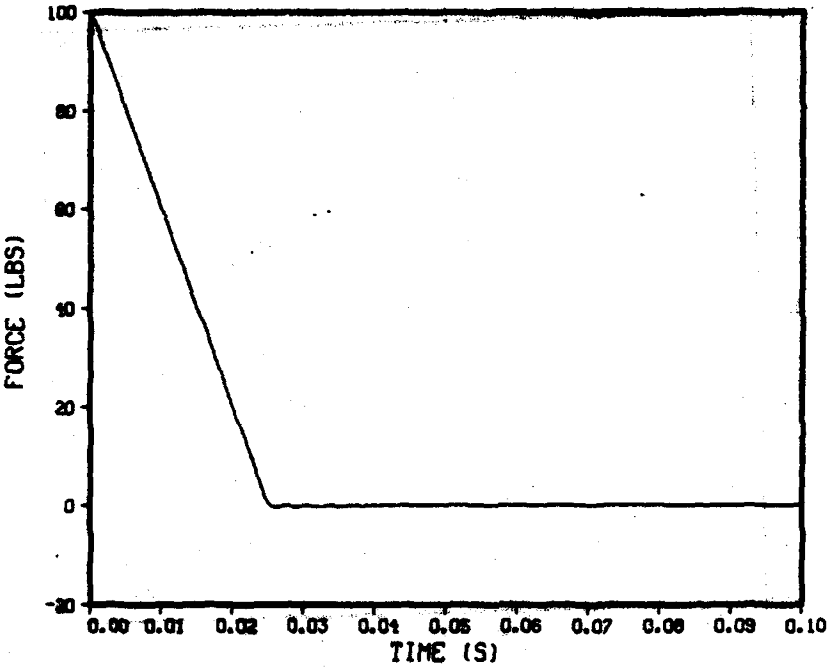

Fig. 3e — Reconstructed input force using only cosine transform

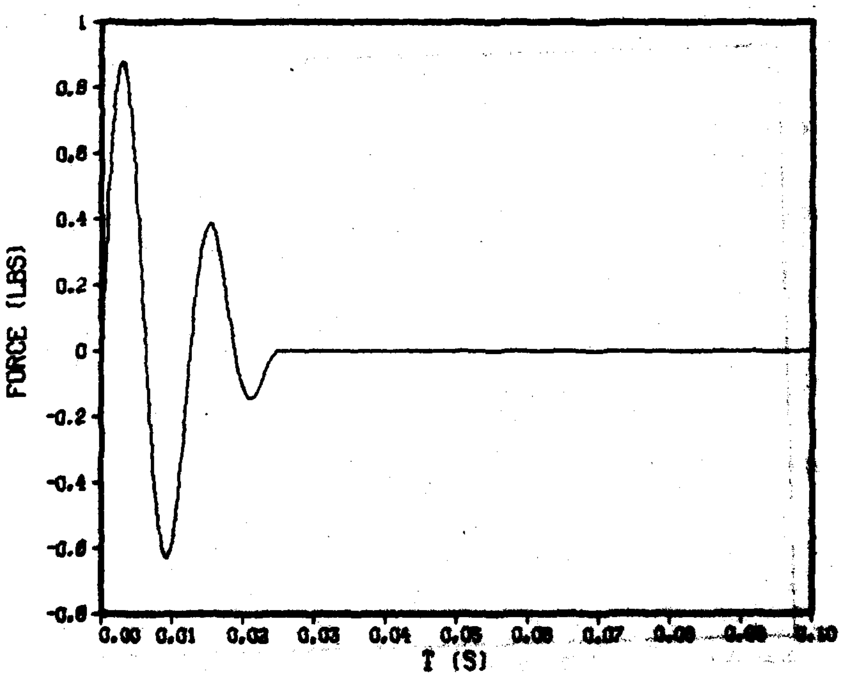

Fig. 4a — Damped sine input force

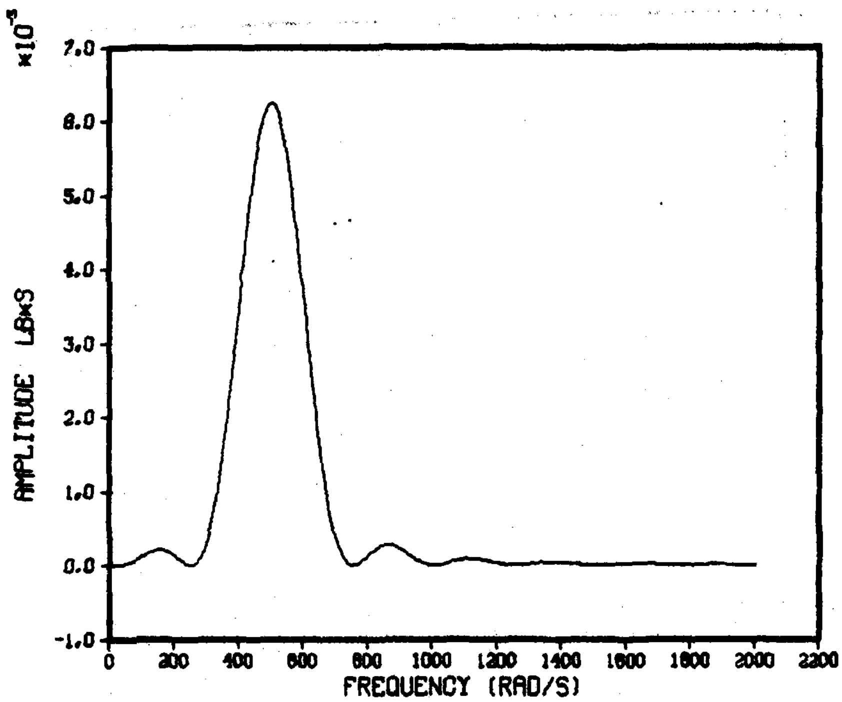

Fig. 4b — Sine transform of reconstructed force

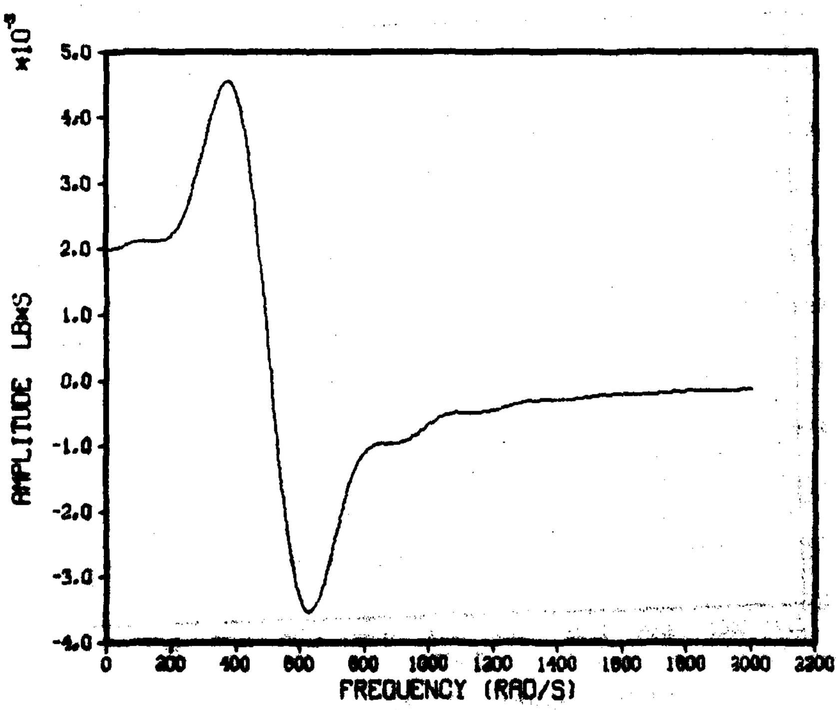

Fig. 4c — Cosine transform of reconstructed force

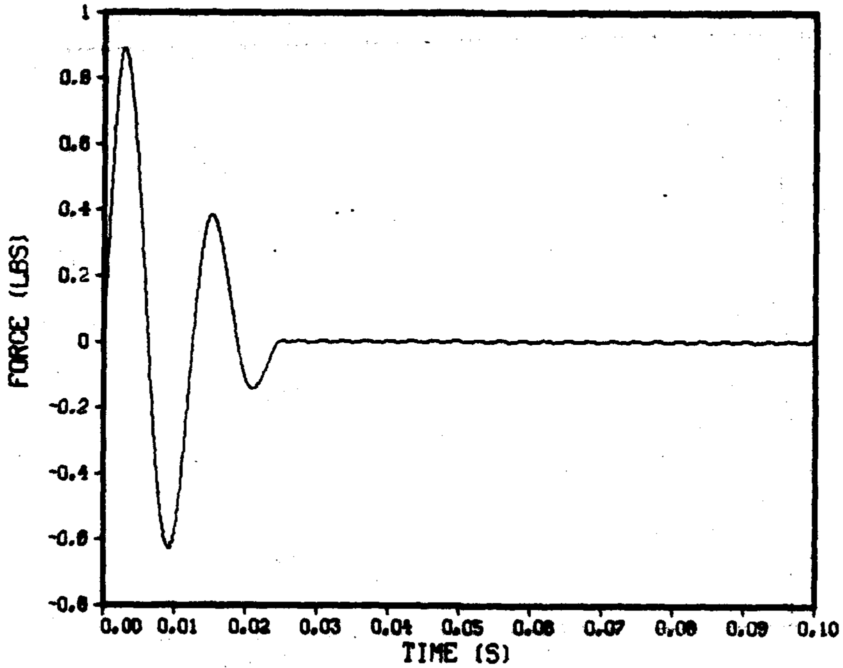

Fig. 4d — Reconstructed input force

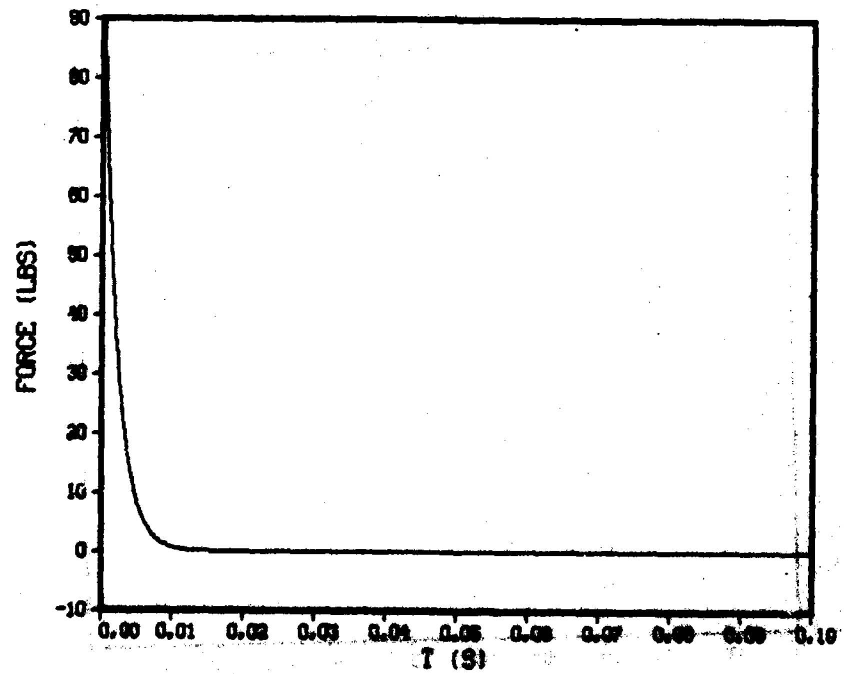

Fig. 5a — Exponential decaying input force

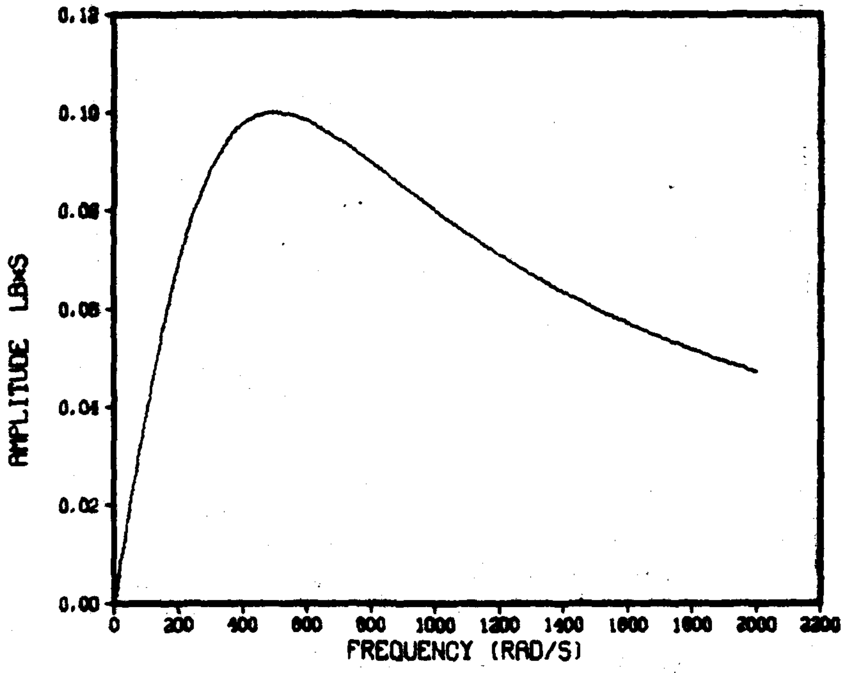

Fig. 5b — Sine transform of reconstructed force

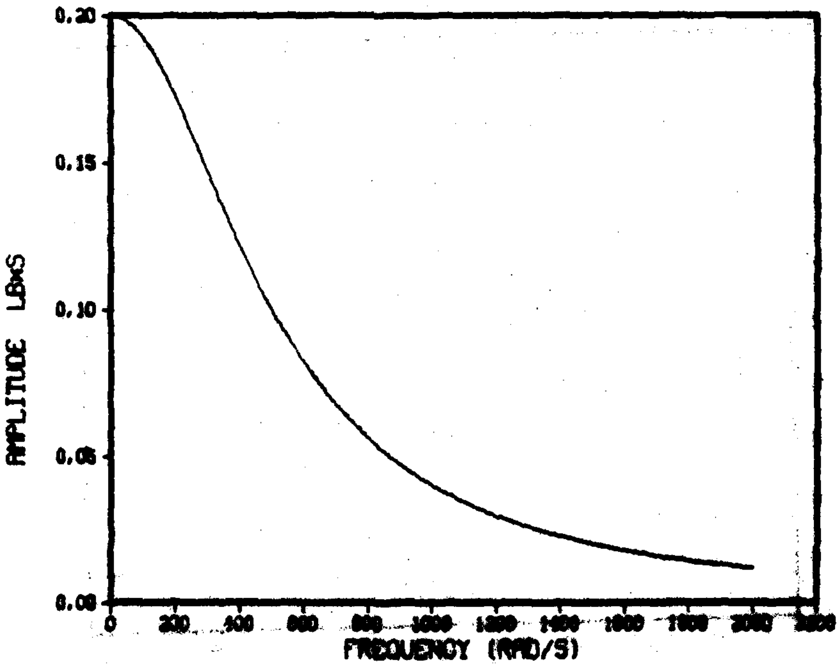

Fig. 5c — Cosine transform of reconstructed force

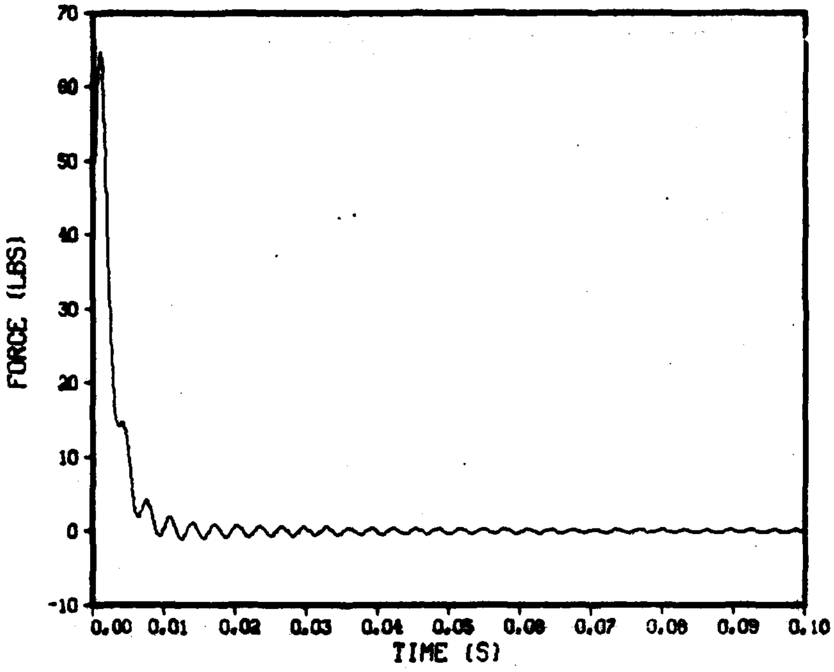

Fig. 5d — Reconstructed input force using both sine and cosine transforms

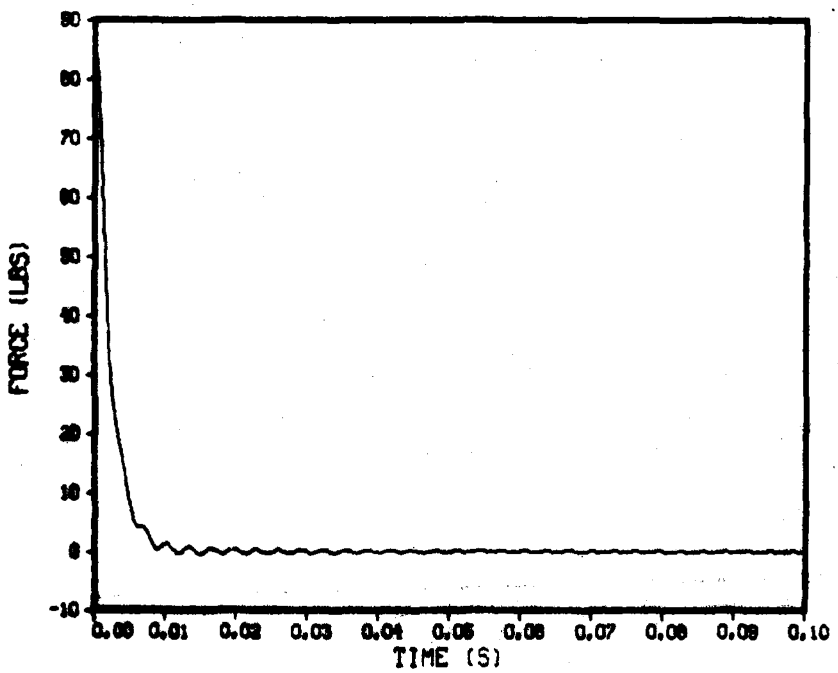

Fig. 5e — Reconstructed input force using only cosine transform